

Temperature cooling in quantum dissipation channel and the corresponding thermal vacuum state*

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Abstract

We examine temperature cooling of optical chaotic light in a quantum dissipation channel with the damping parameter κ . The way we do it is by introducing its thermal vacuum state which can expose entangling effect between the system and the reservoir. The temperature cooling formula is derived, which depends on the parameter κ , by adjusting κ one can control temperature.

1 Introduction

In nature most systems are immersed in their environments, energy exchange between system and its environment always happens, this brings system's dissipation with quantum decoherence [1, 2]. If these systems involve non-negligible correlations amongst their components, quantum memory (non-Markovian) effects cannot be ignored. If the feedback from environment is extremely weak, we can say this process is Markovian, and its dynamics described by the master equation or the associated Langevin or Fokker-Planck equations [7, 8]. The Lindblad equation is the most general form for a Markovian master equation, and it is very important for the treatment of irreversible and non-unitary processes, from dissipation [4] and decoherence [9] to the quantum measurement process [6, 1].

In quantum optics and quantum statistics theory, a damping harmonic oscillator in thermal bath is one of the most famous dissipation model. Its dynamics of this model can be described by the Lindblad master equation. The associated amplitude damping mechanism of system in physical processes is governed by the following master equation [7, 8]

$$\frac{d\rho(t)}{dt} = \kappa (2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \quad (1)$$

where ρ is the density operator of the system, κ is the rate of decay, a, a^\dagger are boson annihilation and creation operator, respectively, $[a, a^\dagger] = 1$. Such an equation can be conveniently solved by virtue of the entangled state representation [10], and the solution is in so-called Kraus form

$$\rho(t) = \sum_{n=0}^{\infty} \frac{V^n}{n!} e^{-\kappa t a^\dagger a} a^n \rho_0 a^{\dagger n} e^{-\kappa t a^\dagger a}, \quad (2)$$

where ρ_0 is system's initial density operator,

$$V \equiv 1 - e^{-2\kappa t}. \quad (3)$$

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As a common knowledge, during the thermal communication between system and reservoir, excitation and de-excitation processes are influenced by the exchange of energy between them. The loss of energy by the system can occur in two ways: 1). It emits quanta with positive energy $\hbar\omega$ (denoted by an annihilation operator a). 2). By creating holes of particles with positive energy in the reservoir. When the latter process takes place, we say that a hole is created in the reservoir by \tilde{a}^\dagger . The tilde mode \tilde{a} (reservoir mode) is independent of the system's mode a , $[\tilde{a}, \tilde{a}^\dagger] = 1$.

An interesting question that has been overlooked for long is how does the temperature of system change when the system undergoes dissipation? To be concrete, we consider when an optical chaotic field [11], described by the density operator

$$\rho_c = \left(1 - e^{-\frac{\hbar\omega}{kT}}\right) e^{-\frac{\hbar\omega}{kT} a^\dagger a}, \quad (4)$$

here $\beta = \frac{1}{kT}$ (k is the Boltzmann constant, T is temperature), undergoes amplitude damping described by Eq. (1), then how is its temperature evolves with time?

In order to answer this question we recall the Thermal Field Dynamics (TFD) theory of Takahashi and Umezawa [3] for converting the statistical average $\text{Tr}(A\rho)$ at nonzero temperature T into equivalent pure state expectation value they introduced the state in a doubled freedom Fock space

$$\sec h\theta \exp[a^\dagger \tilde{a}^\dagger \tanh\theta] |0\tilde{0}\rangle \equiv |0(\beta)\rangle \quad (5)$$

such that

$$\langle 0(\beta) | A | 0(\beta) \rangle = \text{Tr}(A\rho), \quad (6)$$

where the vacuum state $|0\tilde{0}\rangle$ is annihilated by either a or \tilde{a} . The parameter

$$\tanh\theta = \exp\left(-\frac{\hbar\omega}{2kT}\right), \quad (7)$$

is determined by comparing the Bose-Einstein distribution

$$\text{Tr}(\rho_c a^\dagger a) = \left[e^{\hbar\omega/kT} - 1\right]^{-1} \equiv \bar{n}, \quad (8)$$

with the expectation value of the photon number operator in $|0(\beta)\rangle$

$$\langle 0(\beta) | a^\dagger a | 0(\beta) \rangle = \sinh^2\theta. \quad (9)$$

The reason we tackle with $|0(\beta)\rangle \langle 0(\beta)|$ lies in that partial tracing over its tilde-mode will lead to ρ_c (this will be proved in Sec. 2), i.e.

$$\tilde{\text{Tr}}[|0(\beta)\rangle \langle 0(\beta)|] = \rho_c = \left(1 - e^{-\frac{\hbar\omega}{kT}}\right) e^{-\frac{\hbar\omega}{kT} a^\dagger a}, \quad (10)$$

then when we take $|0(\beta)\rangle \langle 0(\beta)|$ as ρ_0 and substitute it into Eq. (2)

$$\rho_c(t) = \sum_{n=0}^{\infty} \frac{V^n}{n!} e^{-\kappa t a^\dagger a} a^n |0(\beta)\rangle \langle 0(\beta)| a^{\dagger n} e^{-\kappa t a^\dagger a}, \quad (11)$$

we will have

$$\begin{aligned} \tilde{\text{Tr}}[\rho_c(t)] &= \sum_{n=0}^{\infty} \frac{V^n}{n!} e^{-\kappa t a^\dagger a} a^n [\tilde{\text{tr}} |0(\beta)\rangle \langle 0(\beta)|] a^{\dagger n} e^{-\kappa t a^\dagger a} \\ &= \sum_{n=0}^{\infty} \frac{T'^n}{n!} e^{-\kappa t a^\dagger a} a^n \rho_c a^{\dagger n} e^{-\kappa t a^\dagger a}, \end{aligned} \quad (12)$$

thus $\tilde{\text{Tr}}[\rho_c(t)]$ will present the correct dissipation evolution law of the chaotic field. Noting that $\rho_c(t)$ itself includes not only the information of the system, but also of the reservoir, and we can also see how the reservoir evolves accompanying the system's dissipation. Moreover, since the temperature effect is manifest through the structure of ρ_c , we also investigate how system's dissipation accompanies the temperature variation.

2 Partial tracing over the tilde-mode of $|0(\beta)\rangle\langle 0(\beta)|$

Using the coherent state representation of the tilde-mode

$$\int \frac{d^2 z}{\pi} |\tilde{z}\rangle \langle \tilde{z}| = 1, \quad \tilde{a} |\tilde{z}\rangle = z |\tilde{z}\rangle, \quad (13)$$

where

$$|\tilde{z}\rangle = \exp \left[-\frac{|z|^2}{2} + z \tilde{a}^\dagger \right] |\tilde{0}\rangle, \quad (14)$$

we have

$$\begin{aligned} \tilde{\text{Tr}} [|0(\beta)\rangle\langle 0(\beta)|] &= \tilde{\text{Tr}} \left[\int \frac{d^2 z}{\pi} |\tilde{z}\rangle \langle \tilde{z}| |0(\beta)\rangle\langle 0(\beta)| \right] \\ &= \sec h^2 \theta \int \frac{d^2 z}{\pi} \langle \tilde{z}| e^{a^\dagger z^* \tanh \theta} |0\tilde{0}\rangle \langle 0\tilde{0}| e^{az \tanh \theta} |\tilde{z}\rangle. \end{aligned} \quad (15)$$

Then using $\langle \tilde{z}| \tilde{0}\rangle = \exp(-|z|^2/2)$, the normal ordering of $|0\rangle\langle 0|$

$$|0\rangle\langle 0| = : e^{-a^\dagger a} : , \quad (16)$$

and

$$e^{\lambda a^\dagger a} = : \exp[(e^\lambda - 1) a^\dagger a] :$$

we have

$$\tilde{\text{Tr}} [|0(\beta)\rangle\langle 0(\beta)|] = \sec h^2 \theta \int \frac{d^2 z}{\pi} : e^{-|z|^2 + a^\dagger z^* \tanh \theta + az \tanh \theta - a^\dagger a} : = \sec h^2 \theta : e^{a^\dagger a (\tanh^2 \theta - 1)} : \quad (17)$$

and noting $\tanh \theta = \exp(-\frac{\hbar\omega}{2kt})$

$$\tilde{\text{Tr}} [|0(\beta)\rangle\langle 0(\beta)|] = [1 - \exp(-\frac{\hbar\omega}{2kt})] \exp(-\frac{\hbar\omega}{2kt} a^\dagger a) = \rho_c. \quad (18)$$

Note that the partial trace over mode a for $|0(\beta)\rangle\langle 0(\beta)|$ is $\text{Tr} [|0(\beta)\rangle\langle 0(\beta)|] = (1 - e^{-\frac{\hbar\omega}{kT}}) e^{-\frac{\hbar\omega}{kT} \tilde{a}^\dagger \tilde{a}}$, since in Eq. (5) $|0(\beta)\rangle$ is symmetric with respect to \tilde{a}^\dagger and a^\dagger .

Remarkably, the thermo vacuum state can be rewritten as

$$|0(\beta)\rangle = \sec h \theta \exp[a^\dagger \tilde{a}^\dagger \tanh \theta] |0\tilde{0}\rangle = S(\theta) |0\tilde{0}\rangle \quad (19)$$

where

$$S(\theta) = \exp[\theta(a^\dagger \tilde{a}^\dagger - a \tilde{a})] \quad (20)$$

is in form like a two-mode squeezing operator [12], so $S(\theta)$ is named thermo squeezing operator which squeezes the vacuum state $|0\tilde{0}\rangle$ at zero-temperature to the thermo vacuum state $|0(\beta)\rangle$ at finite temperature T . Because a two-mode squeezed state is an entangled state, so $|0(\beta)\rangle$ can be considered an entangled state in which the system's mode a^\dagger entangles with the tilde mode \tilde{a}^\dagger . Since they are entangled, the dissipation of system will affect its environment, as we shall show in the next section.

3 Evolution of $|0(\beta)\rangle\langle 0(\beta)|$ in dissipation channel

Using

$$a^n |0(\beta)\rangle = a^{n-1} \sec h \theta [a^n, e^{a^\dagger \tilde{a}^\dagger \tanh \theta}] |0\tilde{0}\rangle = (\tilde{a}^\dagger \tanh \theta)^n |0(\beta)\rangle, \quad (21)$$

we obtain

$$\rho_c(t) = \sum_{n=0}^{\infty} \frac{V^n \tanh^{2n} \theta}{n!} e^{-\kappa t a^\dagger a} \tilde{a}^{\dagger n} |0(\beta)\rangle\langle 0(\beta)| \tilde{a}^n e^{-\kappa t a^\dagger a} \quad (22)$$

$$= \sec h^2 \theta \sum_{n=0}^{\infty} \frac{V^n \tanh^{2n} \theta}{n!} e^{-\kappa t a^\dagger a} \tilde{a}^{\dagger n} e^{a^\dagger \tilde{a}^\dagger \tanh \theta} |0\tilde{0}\rangle\langle 0\tilde{0}| e^{a \tilde{a} \tanh \theta} \tilde{a}^n e^{-\kappa t a^\dagger a} \quad (23)$$

Then using

$$e^{-\kappa t a^\dagger a} a^\dagger e^{\kappa t a^\dagger a} = e^{-\kappa t} a^\dagger \quad (24)$$

we obtain

$$\rho_c(t) = \sec h^2 \theta \sum_{n=0}^{\infty} \frac{V^n \tanh^{2n} \theta}{n!} \tilde{a}^{\dagger n} e^{-\kappa t a^\dagger \tilde{a}^\dagger \tanh \theta} |0\tilde{0}\rangle \langle 0\tilde{0}| e^{e^{-\kappa t} a \tilde{a} \tanh \theta} \tilde{a}^n. \quad (25)$$

Further, by using the normal product form

$$|0\tilde{0}\rangle \langle 0\tilde{0}| =: e^{-a^\dagger a - \tilde{a}^\dagger \tilde{a}}:, \quad (26)$$

we can make summation in Eq. (25) and derive the compact form of $\rho(t)$,

$$\begin{aligned} \rho_c(t) &= \sec h^2 \theta: \sum_{n=0}^{\infty} \frac{V^n \tanh^{2n} \theta}{n!} \tilde{a}^{\dagger n} \tilde{a}^n e^{-\kappa t a^\dagger \tilde{a}^\dagger \tanh \theta} e^{e^{-\kappa t} a \tilde{a} \tanh \theta - a^\dagger a - \tilde{a}^\dagger \tilde{a}}: \\ &= \sec h^2 \theta: \exp\{\tilde{a}^\dagger \tilde{a} (1 - e^{-2\kappa t}) \tanh^2 \theta + e^{-\kappa t} \tanh \theta (a^\dagger \tilde{a}^\dagger + a \tilde{a}) - a^\dagger a - \tilde{a}^\dagger \tilde{a}\}: \\ &= \sec h^2 \theta e^{e^{-\kappa t} a^\dagger \tilde{a}^\dagger \tanh \theta} |0\rangle \langle 0| \exp\{\tilde{a}^\dagger \tilde{a} \ln[(1 - e^{-2\kappa t}) \tanh^2 \theta]\} e^{e^{-\kappa t} \tanh \theta a \tilde{a}} \end{aligned} \quad (27)$$

where $\exp\{\tilde{a}^\dagger \tilde{a} \ln[(1 - e^{-2\kappa t}) \tanh^2 \theta]\}$ indicates that the reservoir is in a chaotic field of the tilde mode, no more in $|\tilde{0}\rangle \langle \tilde{0}|$, this is because the system mode and the reservoir mode are entangled, the dissipation of system certainly affects the reservoir. From (27) we can realize how a pure thermo vacuum state evolves into the mixed state during the dissipation process, that is, not only the squeezing parameter $\tanh \theta \rightarrow e^{-\kappa t} \tanh \theta$, but also $|0\tilde{0}\rangle \langle 0\tilde{0}|$ evolves into $|0\rangle \langle 0| \exp\{\tilde{a}^\dagger \tilde{a} \ln[(1 - e^{-2\kappa t}) \tanh^2 \theta]\}$, i.e., in the process the thermo squeezing effect decreases while the reservoir-mode vacuum becomes chaotic.

4 Partial tracing over the tilde-mode of $\rho_c(t)$

Now we perform partial trace over the tilde-mode of $\rho_c(t)$, using (13), (16-18) and (27) we have

$$\begin{aligned} \tilde{\text{Tr}}[\rho_c(t)] &= \sec h^2 \theta \tilde{\text{Tr}} \left[\frac{d^2 z}{\pi} |\tilde{z}\rangle \langle \tilde{z}| \exp[e^{-\kappa t} a^\dagger \tilde{a}^\dagger \tanh \theta] |0\rangle \langle 0| \exp\{\tilde{a}^\dagger \tilde{a} \ln[(1 - e^{-2\kappa t}) \tanh^2 \theta]\} \exp[a \tilde{a} e^{-\kappa t} \tanh \theta] \right] \\ &= \sec h^2 \theta \int \frac{d^2 z}{\pi}: \exp\left\{|z|^2 [(1 - e^{-2\kappa t}) \tanh^2 \theta - 1] + e^{-\kappa t} (a^\dagger z^* + a z) \tanh \theta - a^\dagger a\right\}: \\ &= \frac{1}{1 + e^{-2\kappa t} \sinh^2 \theta} \exp \left[a^\dagger a \ln \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta} \right]. \end{aligned}$$

When $t = 0$, it becomes the original chaotic state. By identifying

$$\frac{e^{-\kappa t} \tanh \theta}{\sqrt{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta}} = \tanh \theta', \quad (28)$$

then

$$\frac{1}{1 + e^{-2\kappa t} \sinh^2 \theta} = \sec h^2 \theta' \quad (29)$$

and we can express

$$\tilde{\text{Tr}}[\rho_c(t)] = \sec h^2 \theta' \exp[a^\dagger a \ln \tanh^2 \theta'], \quad (30)$$

which explains that the system is still in a chaotic state but with new parameter θ' . In similar to Eq. (7), by identifying

$$\ln \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta} = -\frac{\hbar \omega}{k T'}, \quad (31)$$

we see that system is now at the temperature

$$T' = -\frac{\hbar \omega}{k \ln \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta}}. \quad (32)$$

Due to

$$1 - (1 - e^{-2\kappa t}) \tanh^2 \theta > e^{-2\kappa t}, \quad (33)$$

so

$$\ln \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta} < 0, \quad T' > 0. \quad (34)$$

Moreover, since

$$\tanh^2 \theta > \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta}, \quad (35)$$

$$-\frac{1}{\ln \tanh^2 \theta} > \frac{-1}{\ln \frac{e^{-2\kappa t} \tanh^2 \theta}{1 - (1 - e^{-2\kappa t}) \tanh^2 \theta}}, \quad (36)$$

and in reference to Eq. (7), $\ln \tanh^2 \theta = -\frac{\hbar\omega}{kT}$, $T = -\frac{\hbar\omega}{k \ln \tanh^2 \theta}$, we see

$$T > T' \quad (37)$$

which states that during the damping process the system's temperature decreases, the rate of decreasing can be controlled by adjusting the damping rate κ .

In summary, we have examined temperature cooling of optical chaotic light in a quantum dissipation channel with the damping parameter κ . The way we do it is by introducing its thermal vacuum state which can expose entangling effect between the system and the reservoir. The temperature cooling formula (32) is derived, which depends on the parameter κ , by adjusting κ one can control temperature.

References

- [1] C. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin), (2000).
- [2] J. Ankerhold, in Irreversible Quantum Dynamics, F. Benatti and T. Floreanini Eds., *Lecture Notes in Physics*, Vol. 622, (Springer, Berlin), (2003).
- [3] Y. Takahashi and H. Umezawa, *Collective Phenomena* **2**, 55 (1975). Memorial Issue for H. Umezawa, *Int. J. Mod. Phys. B* **10**, memorial issue and references therein (1996).
- [4] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford), (2002).
- [5] R. Loudon and P. L. Knight, *J. Mod. Optics* **34**, 709 (1987).
- [6] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press), (1995).
- [7] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press), (1998).
- [8] M. Orszag, *Quantum Optics* (Springer Press), (2000).
- [9] H. Y. Fan and L. Y. Hu, *Mod. Phys. Lett B* **22**, 2435 (2008).
- [10] H. Y. Fan and J. R. Klauder, *Phys. Rev. A* **49**, 704 (1994); H. Y. Fan and B. Z. Chen, *Phys. Rev. A* **53**, 2948 (1996).
- [11] H. Y. Fan, J. Zhou, X. X. Xu, and L. Y. Hu, *Chin. Phys. Lett.* **28**(4), 040302 (2011).
- [12] C. C. Wang, H. Y. Fan, *Chin. Phys. Lett.* **27**(11), 110302 (2010).